SPECIFIC FEATURES OF THE PROCESSES OF DRYING AND EVAPORATION IN THE ABSENCE OF AN ANALOGY BETWEEN EXTERNAL HEAT AND MASS EXCHANGE

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UDC 66.047:54.149

The impossibility of constancy of evaporation and wet-bulb temperatures in the absence of analogy between the processes of interphase heat and mass transfer has been shown. It has been noted that, in the general case, the wet-bulb and adiabatic saturation temperatures depend on the process hydrodynamics and their values do not always coincide.

In the theory of drying processes, one of the parameters characterizing the state of moist air is the so-called wet-bulb temperature t_w, which is considered to be a thermodynamic parameter [1–4]. And it is assumed thereby that the evaporation temperature in the constant-drying-rate period is constant and the wet-bulb temperature and the temperature of adiabatic saturation of the air are equal. Since a theoretical justification of the above assumptions is absent, we will attempt to define more exactly these concepts and redetermine their validity range. To this end, we turn to the thermodynamics of the process of interphase interaction.

Let us represent the drying process as a series of sequential elementary acts of interaction between some quantity of moist air with an absolutely dry mass L and a moist material having a dry mass G. Let the interaction occur at a constant total pressure. Then, in the chosen gas + material system the change in enthalpy within each elementary act according to the first law of thermodynamics can be given in the form

$$L dI_g + G dI_{sol} = dQ_{ad} . \tag{1}$$

The additional heat supplied to the chosen system due to the external heat sources (including the environment loss) is given as follows:

$$dQ_{ad} = - G q_{ad} du . \tag{2}$$

We represent the quantity q_{ad} as the sum

$$q_{ad} = q_{ad.g} + q_{ad.sol} . \tag{3}$$

In the case of the absence of external heat sources (adiabatic process), the enthalpy of the chosen system according to (1) should remain constant. At a contact, equilibrium between the phases can be attained when the air becomes saturated (upon evaporation of only the free liquid). Let us call the temperature that becomes stable under these conditions the adiabatic saturation temperature of the gas $t_{a.sat}$.

Let us represent the change in the specific enthalpy of the solid phase in one elementary act in the form

$$dI_{sol} = (c_{liq}\theta_s - q_b)\, du + c_{sol} d\theta . \tag{4}$$

Here the expression between brackets takes into account the physical heat of a liquid with temperature θ_s abstracted from the material due to the water evaporation ($du < 0$), and the last term takes into account the heat received from the gas phase and the external sources and expended in heating the material. The physical heat of a bound liquid is
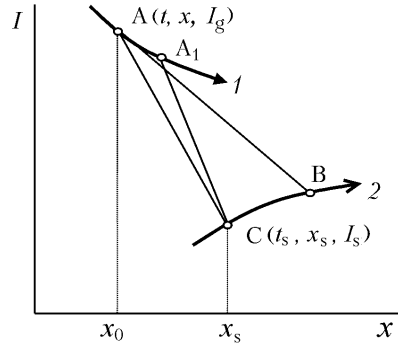
Fig. 1. Interaction between the gas and solid phases in the I—x diagram of moist air.

smaller than the heat of a free liquid $c_{liq}\theta_s$ by the binding energy value q_b [5], since the bonding of the liquid to the dry skeleton (wetting) of the material is accompanied by the release into the environment of a heat corresponding to this energy.

Solving (1)–(4) in view of the material balance equation in an elementary act

$$\frac{G}{L}\,du = -\,dx\,, \tag{5}$$

we can express the change in the gas enthalpy in terms of the solid-phase parameters:

$$dI_g = c_{liq}\theta_s dx - q_b dx + c_{sol}\frac{d\theta}{du}\,dx + q_{ad}dx\,. \tag{6}$$

Here the parameters with a subscript s pertain to the solid-phase surface and those without subscripts — to the average-volume quantities.

Note that the adiabatic saturation temperature of the gas introduced above is more general compared to the temperature used by some authors ($t_{a.sat}$), defined for the hypothetical case of the absence of heat expenditures for heating the material (at θ = const). And Eq. (6) thereby takes on the form $dI_g = c_{liq}\theta_s dx$, where $\theta_s = t_{a.sat}$, i.e., the material should be preheated to the temperature $t_{a.sat}$. For now, however, let us leave the question of the fulfillment of the condition θ_s = const open.

We shall use in our further reasoning the I—x diagram of moist air. Let curve 1 (Fig. 1) characterize the change in the gas parameters in the process of contact with the solid phase and curve 2 — in the parameters of the solid-phase surface, more precisely, the parameters of the gas that is at equilibrium with the surface ($\theta_s = t_s$). Suppose that at a given instant of time a gas with parameters characterized by the point A is in contact with a material having parameters of the point C. Connecting these points, we obtain the phase-contact line AC. Its angular coefficient is equal to the relation $(I_s - I)/(x_s - x)$. Consider also the tangent to curve 1 at the point A in question. Its equation is easy to derive from (6):

$$\frac{dI_g}{dx} = c_{liq}\theta_s - q_b + c_{sol}\frac{d\theta}{du} + q_{ad}\,. \tag{7}$$

The tangent will intersect the curve at some point B. It is essential to elucidate the relation between the straight lines AB and AC. The fact is that in many works on the kinetics of the processes of evaporation and convective drying the assumption about their coincidence is used, i.e., it is assumed that the contact line and the tangent are one and the same thing [2–4]. Despite the wide use of this assumption in the calculations (calculations of scrubbers, water-cooling towers, and other heat exchangers of mixing), it is far from evident. In the given case, the equality

$$\frac{dI_g}{dx} = \frac{I_s - I_g}{x_s - x}\,. \tag{8}$$

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should take place.

The expressions for the gas-phase enthalpy (total mass and mass that is at equilibrium with the surface) in the classical representation have the form

$$I_g = r_0 x + c_{d.g} t + c_v x t \,, \tag{9a}$$

$$I_s = r_0 x_s + c_{d.g} t_s + c_v x_s t_s \,. \tag{9b}$$

Differentiating (9a) with respect to x, we get

$$\frac{dI_g}{dx} = c_g \frac{dt}{dx} + r_0 + c_v t \,. \tag{10}$$

Then from (9b) and (9a), upon a number of manipulations, we have

$$\frac{I_s - I_g}{x_s - x} = c_g \frac{t_s - t}{x_s - x} + r_0 + c_v t_s \,, \tag{11}$$

where $c_g = c_{d.g} + c_v x$.

To clarify the conditions under which (8) is fulfilled, we turn to the general balance relations. Let us write the heat balance for the gas phase in one elementary act in the form

$$L dI_g = L\,[(r_0 + c_v t)\,dx + c_g dt] = dQ_s + dQ_v + dQ_{ad.g} \,. \tag{12}$$

Here dI_g denotes the change in the gas-phase enthalpy in one elementary act, dQ_s is the heat transferred by the gas to the solid phase through the interface F:

$$dQ_s = -\,\alpha\,(t - t_s)\,F d\tau \,; \tag{13}$$

dQ_{vapor} is the heat supplied by the vapor that enters the gas phase with temperature t_s:

$$dQ_v = I_v dW = (r_0 + c_v t_s)\,dW \,; \tag{14}$$

dW is the quantity of released vapor:

$$dW = L dx = \beta\,(x_s - x)\,F d\tau \,; \tag{15}$$

$dQ_{ad.g}$ is the additional heat supplied from the outside to the gas phase.

In a number of cases, in considering the heat balance in the gas phase, one takes into account the additional decrease in its enthalpy due to the expansion of released vapor from pressure p_s to p [3], for which it is necessary to introduce into (12) an additional term Q_{exp} with a negative sign. In this case, however, it would also be necessary to introduce such a term into (6), which contradicts (1) and (4). Consequently, the work of vapor expansion does not necessarily lead to a decrease in the gas enthalpy. This can be explained by the fact that the released vapor, expanding, simultaneously delivers the work of compression of the vapor present in the air.

The joint solution of (12)–(15) yields

$$c_g \frac{dt}{dx} = -\frac{\alpha}{\beta} \frac{t - t_s}{x_s - x} + c_v\,(t_s - t) + q_{ad.g} \,. \tag{16}$$

Substituting (16) into (10), we obtain in the general case

$$\frac{dI_g}{dx} = -\frac{\alpha}{\beta} \frac{t - t_s}{x_s - x} + c_v t_s + r_0 + q_{ad.g} \,. \tag{17}$$

Equating (7) to (17) taking into account (3), we have

$$c_{sol} \frac{d\theta}{du} = r - \frac{\alpha}{\beta} \frac{t - t_s}{x_s - x} - q_{ad.sol} \,, \tag{18}$$

where $r = r_0 - c_{liq}\theta_s + c_v\theta_s + q_b$ is the moisture evaporation heat at temperature θ_s. Equation (18) relates the parameters of the contacting phases to the temperature coefficients of drying $d\theta/du$ and, therefore, can serve as the basic equation of material heating.

From comparison of (17) and (11) it follows that in the general case the coincidence of the contact line with the tangent in the I—x diagram is absent.

We shall further consider the process of evaporation of the free liquid in the absence of additional heat sources and internal gradients ($\theta_s = \theta$), since this process in the drying theory is one of the basic ones. For this case, $q_b = q_{ad.g} = q_{ad.sol} = 0$. Comparing (17) and (11) at $q_{ad.g} = 0$, note that for the tangent to coincide with the contact line, the ratio between the interphase heat and mass transfer coefficients should satisfy the expression

$$\alpha/\beta = c_g \,. \tag{19}$$

which is a kind of the known Lewis relation [5–8]. The latter holds only in the presence of a complete analogy between the processes of interphase heat and mass transfer (coincidence of the thermal and diffusion boundary layers), which requires equality of the Prandtl (Pr) and Schmidt (Sc) numbers [3, 5, 6]. However, since in the majority of cases this relation does not hold, there is no reason to argue that the tangent and the contact line coincide.

Consider some consequences of this fact.

First, from here it follows that in the drying process the material temperature cannot be constant in principle, even upon evaporation of the free liquid. Indeed, let the parameters of the contacting phases at an arbitrary instant of time be given by the contact line AC (Fig. 1). Let its angular coefficient $(t - t_s)/(x - x_s)$ have such a value that, according to Eq. (18), the condition $d\theta = 0$ is met, i.e., there is no increase in the material temperature. If the change in the parameters occurred on the AC line, then the angular coefficient of the contact line would remain unaltered and we could speak of constancy of the evaporation temperature.

Actually, the change in the gas parameters occurs on the AB line. Let, at the next instant of time, the state of the gas be represented by the point A_1. It can easily be seen that the angular coefficient of the contact line A_1C will change. Then the second term in Eq. (18) will also change (at the same values of u, θ, t_s, x_s, r), the heat balance is disturbed ($d\theta \neq 0$), and the material temperature will begin to change ("float").

Since usually for water vapor $\alpha/\beta < c_g$, the above inequality decreases the negative term in (18) compared to the first term in (11). Consequently, the angular coefficient of the tangent is larger than the angular coefficient of the contact line, and the point B is positioned to the right of the point C. This gives reason to argue that the evaporation temperature will gradually increase, approaching the adiabatic saturation temperature.

Second, the absence of a constant evaporation temperature according to (17) at $t_s \neq$ const leads to a curvature of the line of change in the gas enthalpy in the first period of drying, i.e., strictly speaking, this line is not straight. Therefore, the use in many sources for the constant-drying-rate period of the expression $(I_2 - I_1)/(x_2 - x_1) = c_{liq}t_w$ is not accurate enough, and integration of (17) taking into account (18) seems to be more correct.

Third, the use in the calculations of a constant evaporation temperature ($t_s = $ const) or the coincidence of the contact line with the tangent automatically means the assumption of an analogy between the processes of interphase heat and mass transfer [9], while authors can deny the latter.

The quantitative estimate of the range of change in the evaporation temperature in the first period was made by numerical integration of the system of ordinary differential equations (16), (18), (5). This system makes it possible to calculate the change in the parameters of contacting phases for constant quantities of the gas and the solid phase (periodic process) and in the forward flow (continuous process). For the counterflow of phases in (5) one has to re-
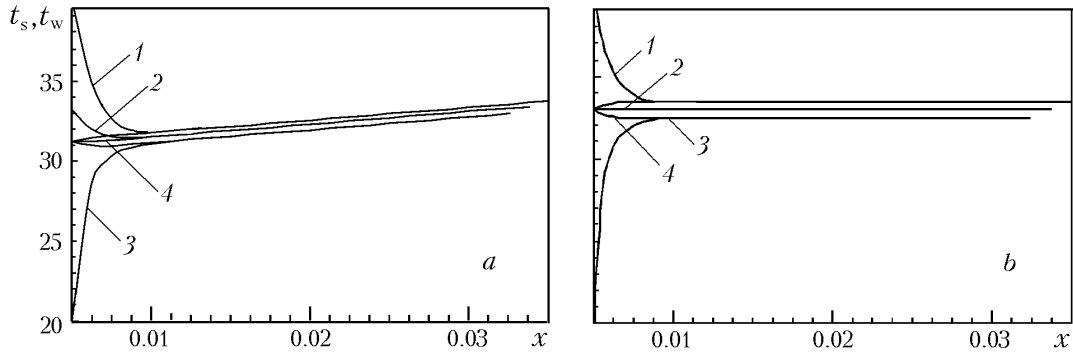
Fig. 2. Change in the evaporation temperature (1–3) and the wet-bulb temperature (4) in coordinates: a) $\alpha/\beta = 0.85c_g$ at the initial temperature of the material θ_0: 1) 40; 2) 31.2; 3) 20°C; b) $\alpha/\beta = c_g$, θ_0: 1) 40; 2) 33; 3) 20°C.

place the negative sign by a positive one. The initial conditions for the periodic process and the forward flow are as follows: $t = t_0$, $\theta = \theta_0$, and $u = u_0$ at $x = x_0$. The results of the calculations are presented in Fig. 2 in the $t_s - x$ coordinates. We used the following input data: $t_0 = 100°C$, $u_0 = 2$ kg/kg, $x_0 = 0.005$ kg/kg, and $L/G = 20$. The internal heat and moisture gradients were assumed to be absent and, therefore, the material temperature coincided with the evaporation temperature ($\theta = t_s = t_{sat}$). The coefficient ratio α/β was taken to be equal to $0.85c_g$ (absence of analogy) and c_g (analogy), and the initial temperature of the moist material was varied from 20 to 40°C.

The evaporation of only the free moisture at the known dependence of $x_s = x_{sat}$ on t_{sat} was investigated. In the case of the absence of analogy (Fig. 2a), as was mentioned above, a smooth increase in the material temperature until equilibrium (adiabatic saturation) is attained is characteristic. Of particular interest is curve 2, pertaining to the case of the material preheated to the adiabatic saturation temperature. Nevertheless, in the process of contact its temperature is ever changing (first it decreases and then increases again, reaching the previous value at equilibrium). Thus, the calculations confirm the impossibility of evaporation-temperature constancy in the case of the absence of analogy even at $\theta_0 = t_{a.sat}$.

Note that the range of change in the evaporation temperature in the example being considered is comparatively small (about 2°C); therefore, in solving temperature problems, the absence of an analogy is practically imperceptible. As for the process of mass transfer, the change in the value of x_{sat} can reach 10%, and this fact cannot be ignored.

A somewhat different picture of evaporation is observed when the Lewis relation holds (Fig. 2b). In this case, the evaporation temperature upon completion of the warming-up period is practically constant, although it depends on the initial temperature of the material and the rate of flow of the phases.

We now turn to the estimation of the adiabatic saturation temperature that can be reached at the end of the process of contact of phases. The given curves show a clear relation between $t_{a.sat}$ (end points of the curve) and the initial temperature of the material. The value of $t_{a.sat}$ can be calculated by the following balance equations at the known dependence of $x_{a.sat}$ on $t_{a.sat}$:

$$Lr\,(x_{a.sat} - x_0) + G\,(c_{sol}t_{a.sat} - c_{sol0}\theta_0) = L\,(c_{g0}t_0 - c_g t_{a.sat})\,, \tag{20}$$

where $c_{sol} = c_{d.sol} + c_{liq}u$ and $c_{sol0} = c_{d.sol} + c_{liq}u_0$.

From relation (20) it follows that $t_{a.sat}$ depends not only on the initial temperature of the material, but also on the flow rate of phases L and G. Independence of the flow rates of phases is observed only in the case of $\theta_0 = t_{a.sat}$, i.e., when a material heated to the required temperature is fed into the chamber. Thus, the adiabatic saturation temperature in the general case is not a thermodynamic parameter of the state of the gas but depends on the quantities θ_0, L, and G. From Fig 2a it also follows that in the absence of an analogy, the evaporation and adiabatic saturation temperatures can coincide in value only at equilibrium. In the presence of an analogy, these quantities are practically identical upon completion of the warming-up period.

Consider now the situation where $G \ll L$, i.e., the moisture mass is negligibly small compared to the air mass. Then the air parameters at contact with phases remain practically unaltered. Under these conditions, the tangent
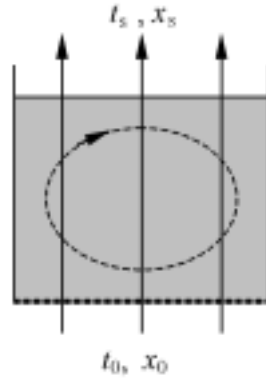
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Fig. 3. On the analysis of the mixed layer.

direction does not play any role (since the gas enthalpy is invariant). Therefore, in principle, evaporation of a liquid with a constant temperature is possible. In the given case, it is called the wet (wetted) bulb temperature and its value can be determined from (18), assuming $d\theta = 0$. For comparison, we give the expressions for calculating t_w:

$$r = \frac{\alpha}{\beta} \frac{t - t_w}{x_{sat} - x},$$

(21a)

$$r = c_g \frac{t - t_w}{x_{sat} - x},$$

(21b)

which correspond to the presence (21b) or absence (21a) of an analogy.

Since $\alpha/\beta < c_g$, the ratio $(t - t_w)/(x_{sat} - x)$ at the same value of r in (21a) should be larger than in (21b). This means that the contact line of (21a) is steeper than that of (21b), i.e., the value of t_w in the first case is smaller. Thus, the wet-bulb temperature in the absence of an analogy is lower than in its presence.

The above conclusion is corroborated by the calculations (see Fig. 2), where curves 4 show the change in the wet-bulb temperature in the process of evaporation. Note that the evaporation and wet-bulb temperatures practically coincide upon completion of the heating period. In so doing, t_w is a state parameter of the gas (independent of the flow rate of phases and the material temperature) only at the very beginning of the process and subsequently changes in accordance with the change in the gas-phase temperature and moisture.

From the calculations, it follows that in the absence of an analogy the wet-bulb temperature (and, accordingly, the evaporation temperature) in the considered examples is $\sim 2^{\circ}$ lower than in the presence of an analogy. This corroborates the above conclusion.

We draw attention to one interesting kind of contact of phases, in which a certain exclusion from the above postulates shows up. Consider a cell with a perfectly mixed layer of the solid phase (Fig. 3) which is blown by a gas stream with constant parameters t_0 and x_0. Typical representatives of such a scheme of interaction of phases are pseudoliquefied and vibromixed layers. Mixing makes it possible to artificially maintain constancy of the solid-phase characteristics as to the layer volume (but not time). We consider the drying process to be periodic.

As a result of the relatively high velocity of the gas as it flows through the layer, the solid-phase parameters can change by only an infinitely small value (unlike the gas parameters). Therefore, it may be assumed that during one elementary act the gas interacts with a material having constant quantities u, θ, x_s, and t_s along its path. As a result, in Eq. (17) only the gas parameters x and t are variable.

Consider the practically important case where the gas at the outlet from the layer comes to equilibrium with the material surface, i.e., the outflowing gas parameters differ but slightly from the surface parameters x_s and t_s.

Since the gas enthalpy is the state parameter, the $I_s - I_0$ difference should be independent of the form of the path covered by the gas. Therefore, integrating (17) with respect to the parameter x from x_0 to x_s, for a fixed instant of time (t_s and x_s under mixing of the solid phase are independent of x, $q_{ad.g} = 0$), we get

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$$\int_{x_0}^{x_s} \frac{dI_g}{dx} dx = I_s - I_0 = \int_{x_0}^{x_s} \frac{\alpha}{\beta} \frac{t - t_s}{x - x_s} dx + (r_0 + c_{vapor}t_s)(x_s - x_0) . \tag{22}$$

Comparing (22) and (11), we obtain

$$\frac{1}{x_s - x_0} \int_{x_0}^{x_s} \frac{\alpha}{\beta} \frac{t - t_s}{x - x_s} dx = c_g \frac{t_0 - t_s}{x_0 - x_s} . \tag{23}$$

Let us also integrate (18) in the same manner:

$$\frac{c_{sol}}{x_s - x_0} \int_{x_0}^{x_s} \frac{d\theta}{du} dx = r - \frac{1}{x_s - x_0} \int_{x_0}^{x_s} \frac{\alpha}{\beta} \frac{t - t_s}{x_s - x} dx . \tag{24}$$

In the case of mixing, the expression on the left side of (24) represents the averaged value of $c_{sol}\dfrac{\overline{d\theta}}{du}$ that characterizes not an individual particle but the whole volume of the material with average-volume temperature and moisture content. Using (23), we get

$$c_{sol} \frac{\overline{d\theta}}{\overline{du}} = r - c_g \frac{t_0 - t_s}{x_s - x_0} . \tag{25}$$

In the case where external heat sources are taken into account, corresponding terms can be introduced into the right side of (25).

    Equation (25) for mixed layers differs from (18) in that it uses not the local values of the temperature t and the moisture content x of the gas but only their values at the inlet and outlet from the layer. Note also the absence of both coefficients of interphase heat and mass transfer. Thus, Eq. (25) permits calculating the material heating throughout the mixed layer mass without taking into account the process of change in the gas state, which is undoubtedly convenient. Therefore, where the processes in mixed layers are concerned, as a curve of the change in the state of the gas as it flows through the layer, the contact line can be used.

    From expression (25) it follows that the wet-bulb temperature in mixed layers is independent of the coefficient ratio α/β and coincides with t_w in the presence of an analogy. Nevertheless, this conclusion does not point to the presence of an analogy in the case under consideration — it is absent because at the local points of the layer Eqs. (16) and (17) hold. It can be shown, however, that throughout the layer volume the consequences of the absence of an analogy are opposite in nature and counterbalance each other.

NOTATION

    c, heat capacity, J/(kg·deg); F, interphase contact surface, m^2; G, dry mass (rate of flow) of the solid phase, kg (kg/sec); I, specific enthalpy, J/kg; L, mass (rate of flow) of dry gas, kg (kg·sec); p, pressure of vapor, Pa; r, moisture evaporation heat, J/kg; r_0, same at 0°C; Q and q, total and specific heat, J and J/kg; t, gas temperature, 0°C; u, moisture content of the material, kg of moisture/kg of dry material; x, moisture of the gas phase, kg/kg of dry air; W, mass of evaporated moisture, kg; α, interphase heat-transfer coefficient, W/(m^2·deg); β, interphase mass-transfer coefficient, kg/(m^2·sec); θ, material temperature, 0°C; τ, time. Subscripts: a.sat, adiabatic saturation; g, gas; ad, additional, ad.g, and ad.sol, additional to the gas and the solid phase, respectively; liq, liquid; w, wet-bulb thermometer; sat, saturated; 0, initial; s, surface; v, vapor; exp, expansion; b, bound; d.g, dry gas; sol, solid phase; d.sol, dry solid phase.

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